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### Claims

We claim:

1. A computer-implemented method for solving integral equations representing wave propagation within a region, the method including:
  - (i) representing an integral equation describing wave propagation as a matrix equation including an impedance matrix and an unknown column vector;
  - (ii) partitioning the impedance matrix on a plurality of levels as a structure of block impedance matrices, each block impedance matrix representing the interaction of two blocks of the grid;
  - (iii) for each pair of blocks, each of size  $b \times b$ , decomposing each block impedance matrix having a rank  $r$  as a product  $UV$  of two matrices  $U$  and  $V$ , where  $U$  has a size  $r$  times  $b$  and  $V$  has a size  $b$  times  $r$ ; and
  - (iv) solving the matrix equation using an iterative method having a number of steps in each of which a column vector is multiplied by  $V$  and the result multiplied by  $U$ .
2. A method according to claim 1 in which the impedance matrix is defined between points on a grid defined in the region.
3. A method according to claim 1 in which the impedance matrix is defined between coefficients of wave expansions for objects defined in the region.
4. A method according to claim 1 in which the matrix equation is for solving Maxwell's equation expressed as a surface integral equation, the method including the steps of forming the matrix equation by

- (a) constructing the surface integral equation with Green's function;
- (b) using basis functions and testing functions to transforming the surface integral equation into the matrix equation.

5. A method according to claim 1 in which the matrix equation is for solving Foldy Lax equations of partial waves, the method including the steps of forming the matrix equation by:

- (a) building a Foldy Lax multiple scattering equation of partial waves for volume scattering;
- (b) transforming the Foldy Lax volume integral equation into the matrix equation.

6. A method according to claim 1, further including calculating the rank of at least one of the block impedance matrices by sampling using coarse sampling from the blocks and forming a matrix of the impedance values between the sampled points, and obtaining the rank by performing an SVD process on the matrix.

7. A method according to claim 4 in which the sampling is performed using boundary sampling.

8. A method according to claim 4 in which the wave equation relates to scattering of a wave and the sampling is performed using coarse-coarse sampling.

9. A method according to claim 1 including determining the rank of of the block impedance matrices and setting up rank tables by

- (a) deriving equivalent circular areas for each of the two blocks;
- (b) performing a cylindrical wave expansion of an equation representing the interaction of the circular areas; and

(c) using an SVD method to determine the rank based on the cylindrical wave expansion.

10. A method according to claim 1 in which the rank of the block impedance matrices of a given problem is extracted from a rank table or from coarse-coarse sampling.

11. A method according to claim 1 in which the wave propagation is in a region including a printed circuit board (PCB).

12. A method according to claim 1 in which the wave propagation is in a region including an integrated circuit (IC) packaging structure.

13. A method according to claim 1 in which the wave propagation is in a region including an interconnect structure.

14. A method according to claim 1 in which the wave propagation is in a region including on-chip structure.

15. A method according to claim 1 in which the wave propagation is in a region including a patch antenna.

16. A method according to claim 1 in which the wave propagation is in a region including a micro-strip antenna.

17. A method according to claim 1 in which the wave propagation is in a region including a rough surface, the rough surface scattering an incident wave.

18. A method according to claim 1 in which the wave propagation is in a region in a nano-material.
19. A method according to claim 1 in which the wave propagation is wave propagation in a region in a metamaterial.
20. A method according to claim 1 in which the wave propagation is in a region in a photonic bandgap structure.
21. A method according to claim 1 in which the wave propagation is wave propagation in a region including a scattering medium consisting of randomly distributed scatterers
22. A method according to claim 1 in which the step of solving the matrix equation includes obtaining a radar cross section of scattering objects in the region.
23. A method according to claim 1 in which the solving the matrix equation will generate an equivalent circuit and analyzing the equivalent circuit with a circuit analysis program.
24. A method according to claim 1 in which after solving the matrix equation , the output is a scattering matrix.
25. A method for determining rank of a block impedance matrix, the block impedance matrix representing the interaction between two blocks of a grid of points or coefficients of wave expansions in scattering a wave transmitted according to a wave equation, the method comprising the steps of:

(a) forming a matrix of the impedance values between sampled points or coefficients; and

(b) subsequently obtaining the rank by performing an SVD process on the sampled matrix;

wherein the sampling is performed using a coarse-coarse sampling.

26. A method according to claim 25 in which the sampled matrix is produced by a selected one of:

(i) coarse sampling points from the blocks, or

(ii) using coarse sampling from the boundaries, or

(iii) using interactions between cylindrical waves in a cylindrical wave expansion.

27. A method for determining rank of a block impedance matrix, the block impedance matrix representing the interaction between two blocks of a grid of points or coefficients of wave expansions of objects in a region of scattering a wave transmitted according to a wave equation, the method comprising the steps of:

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~~(a) deriving equivalent circular areas for each of the two blocks;~~

(b) performing a cylindrical wave expansion of an equation representing the interaction of the circular areas; and

(c) using an SVD method to determine the rank based on the cylindrical wave expansion.

28. A computer apparatus for solving equations representing wave propagation within a region, the apparatus including a processor arranged to perform the steps of:

- (i) representing an integral equation describing wave propagation as a matrix equation including an impedance matrix and an unknown column vector;
- (ii) partitioning the impedance matrix on a plurality of levels as a structure of block impedance matrices, each block impedance matrix representing the interaction of two blocks of the grid;
- (iii) for each pair of blocks, each of size  $b \times b$ , decomposing each block impedance matrix having a rank  $r$  as a product  $UV$  of two matrices  $U$  and  $V$ , where  $U$  has a size  $r$  times  $b$  and  $V$  has a size  $b$  times  $r$ ; and
- (iv) solving the matrix equation based on using iterative method having a number of steps in each of which a column vector is multiplied by the  $V$  and the result multiplied by  $U$ .

29. A record carrier product, readable by a computer apparatus, and carrying computer program instructions which, when executed by a processor of the computer, cause the processor to solve equations representing wave propagation within a region by performing the steps of:

- (i) obtaining a matrix equation of the column vector based on an impedance matrix derived from integral equation describing the wave propagation ;
- (ii) partitioning the impedance matrix on a plurality of levels as a structure of block impedance matrices, each block impedance matrix representing the interaction of two blocks of the grid;
- (iii) for each pair of blocks, each of size  $b \times b$ , decomposing each block impedance matrix having a rank  $r$  as a product  $UV$  of two matrices  $U$  and  $V$ , where  $U$  has a size  $r$  times  $b$  and  $V$  has a size  $b$  times  $r$ ; and

(iv) solving the matrix equation based on using iterative method having a number of steps in each of which a column vector is multiplied by  $V$  and the result multiplied by  $U$ .

30. A record-carrier product according to claim 29 for use in analyzing at least one of: an electronic package structures, an integrated circuit package, a printed circuit board, interconnects, an on-chip structure, a patch antenna, a micro-strip antenna, or a rough surface structure.

31. A record-carrier product according to claim 29 wherein the computer program instructions further cause the processor to determine an equivalent circuit for the electronic package, printed circuit board, IC packaging, and on-chip structure.

32. A record-carrier product according to claim 29, wherein the computer program instructions, when executed by the processor, further cause the processor to compute field solutions and use the field solutions to generate an equivalent circuit and analyze the equivalent circuit with a circuit analysis program.

33. A record-carrier product according to claim 29, wherein the computer program instructions, when executed by the processor, cause the processor to compute field solutions and use the field solutions to output a scattering matrix.

34. A record-carrier product according to claim 33, wherein the computer program instructions, when executed by the processor, cause the processor to compute field equations and use the field solutions to generate an output scattering matrix as input into a microwave network model for analysis of the electronic package.



35. . A record carrier product according to claim 29 which is for use in determining and analyzing the properties of nano materials, metamaterials, photonic bandgap structures, electronic scattering from nanostructures, electronic bandstructures of nanomaterials, and random media structures.